Unit 10 Outline –Convergence and Divergence of Series

Friday 2/7	**Today's Topic:** To use the Direct Comparison Test and Limit Comparison Test for convergence of infinite series

Key Idea:

Direct Comparison Test (DCT)

If $a_n \geq 0$ and $b_n \geq 0$,

1) If $\sum_{n=1}^{\infty} b_n$ converges and $0 \leq a_n \leq b_n$, then $\sum_{n=1}^{\infty} a_n$ ______________________.

2) If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \leq a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ ______________________.

NOTE: You must state/show the inequality when stating the conclusion of the test!!

In-Class Examples: Determine whether the following series converge or diverge:

1. $\sum \frac{1}{2^n + 1}$
2. $\sum_{n=1}^{\infty} \frac{3}{n + \sqrt{n}}$
3. $\sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$
4. $\sum_{n=2}^{\infty} \frac{1}{n^4 - 10}$

Limit Comparison Test

Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$ (N an integer).

1. If $\lim_{n \to \infty} \frac{a_n}{b_n} = c > 0$, then $\sum a_n$ and $\sum b_n$ both converge or both diverge.

2. If $\lim_{n \to \infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

3. If $\lim_{n \to \infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

In-Class Examples: Determine whether the following series converge or diverge:

1. $\sum \frac{2n+1}{n^2 + 2n + 1}$
2. $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$
3. $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$
4. $\sum_{n=1}^{\infty} \frac{1}{n^3 - 2}$

Homework: Worksheet 79

Tuesday 2/11	**Today's Topic:** Learn to use the ratio and root test for convergence or divergence of a series

Key Idea: A series converges if its sequence of partial sums converges

Ratio Test

Let $\sum_{n=1}^{\infty} a_n$ be a series of nonzero terms.

1. $\sum_{n=1}^{\infty} a_n$ converges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$
2. $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$
3. The ratio test is inconclusive if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$

Root Test

1. $\sum_{n=1}^{\infty} a_n$ converges if $\lim_{n\to\infty}\sqrt[n]{|a_n|} < 1$
2. $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty}\sqrt[n]{|a_n|} > 1$
3. The Root Test is inclusive if $\lim_{n\to\infty}\sqrt[n]{|a_n|} = 1$

In-Class Examples:

Determine the following series converge or diverge.

1) $\sum \frac{1}{k!}$

2) $\sum \frac{k}{2^k}$

3) $\sum \frac{n^n}{n!}$

4) $\sum_{n=1}^{\infty} \frac{(2n)!}{4^n}$

5) $\sum \frac{e^{2k}}{k^k}$

6) $\sum_{n=1}^{\infty}\left(\frac{3n+4}{2n}\right)^n$

Homework: Worksheet 81

Wednesday 2/12	**Today's Topic:** Determining the convergence or divergence of alternating series. Approximating error of the sum of an alternating series

Key Ideas:

Alternating Series Test (AST)

If $a_n > 0$, then the alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ converges if both of the following conditions are satisfies:

1) $\lim_{n\to\infty} a_n = 0$

2) $\{a_n\}$ is a decreasing (or Non-increasing) sequence; that is, $a_{n+1} \le a_n$ for all $n > k$, for some $k \in \mathbb{Z}$

Note: This does NOT say that if $\lim_{n\to\infty} a_n \neq 0$ the series DIVERGES by the AST. The AST can ONLY be used to prove convergence. If $\lim_{n\to\infty} a_n \neq 0$, then the series diverges, but by the ***n*th-term** test NOT the AST.

In-Class Examples: Determine the convergence of divergence of each series.

1) $\sum(-1)^{n+1}\frac{n+3}{n(n+1)}$ 2) $\sum(-1)^{n+1}\frac{1}{n}$ 3) $\sum\frac{(-1)^{k+1}}{2k+1}$ 4) $\sum\frac{\cos(n\pi)}{n}$

5) Estimate the sum if the first 5 terms of $\sum(-1)^{n+1}\frac{1}{n}$ are used to estimate *S*. Find the error in the approximation.

6) Estimate the sum if the first 5 terms of $\sum\frac{(-1)^{n-1}}{n!}$ are used to estimate *S*. Find the error in the approximation.

Homework: Worksheet 82

Thursday 2/13	**Today's Topic:** Determine if a series is absolutely convergent, conditionally convergent, or divergent

Key Idea:

If the series $\sum_{n=1}^{\infty}|a_n|$ converges, then $\sum_{n=1}^{\infty}a_n$ also converges.

Such a series is called **absolutely convergent**. Notice that if it converges on its "own," the alternator only allows it to converge more "rapidly".

$\sum_{n=1}^{\infty}a_n$ is **conditionally convergent** if $\sum_{n=1}^{\infty}a_n$ converges but $\sum_{n=1}^{\infty}|a_n|$ diverges.

In-Class Examples: Determine if a series is absolutely convergent, conditionally convergent, or divergent

1) $\sum(-1)^{n+1}\frac{1}{n^2}$ 2) $\sum(-1)^{n+1}\frac{1}{n}$ 3) $\sum(-1)^{n+1}\frac{n}{5n+1}$ 4) $\sum(-1)^{n+1}\frac{n+4}{n^3}$

Homework: Worksheet 83

Friday 2/14	Today’s Topic: Review for the test on series
In-Class Examples: None	
Homework: Worksheet 84	

Tuesday 2/18	Today’s Topic: Review for Thursday’s Test
In-Class Examples: None	
Homework: Worksheet 85	

Wednesday 2/19	Today’s Topic: Review for Thursday’s Test
In-Class Examples: None	
Homework: Worksheet 86	

Thursday 2/20	Today’s Topic: Test on Convergence and Divergence
In-Class Examples: None	
Homework: None	